Mathematical model of river pollution prediction with and without input sources: A particular case of river Tapi (Surat), Gujarat

1Priti V. Tandel,

2Manan A. Maisuria

Department of Mathematics,

Veer Narmad South Gujarat University,

Surat, Gujarat, India.

pvtandel@vnsgu.ac.in1

mananmaisuria.maths21@vnsgu.ac.in2

Abstract:
The present study examines a mathematical model for the examination of pollution transport and diffusion processes in river systems. We have presented a one-dimensional model in this paper, which is represented by differential equations for convection and diffusion that vary with time. Our research focuses on the river's potential for contamination, both in the presence and absence of a particular source of pollution. The objective of the present study is to examine the effects of two distinct input source functions, namely the sinusoidal and exponential forms. We have done research on the diffusion and transport of pollutants, concentrating on the phosphate and nitrate of the Tapi River, in order to verify the model. To get the answers, the Reduced Differential Transform Method (RDTM) is used. The convergence of the solution function is assessed for each instance derived from RDTM in order to determine the accuracy of the solution. In every case, the pollutant concentration at different times and distances has been shown numerically. 3D graphs are used to compare the pollution levels with and without sources. The method detailed in this research paper, which utilizes a one-dimensional pollution model to forecast river pollution, applies to additional rivers.

1 Introduction

The public's concern for environmental preservation has grown significantly in recent years. This is owing to the fact that future generations will need to have access to resources. The discharge of pollutants into surface waterways is the root of many issues. Pollutants are being discharged into many rivers, particularly those that run through densely populated regions (Ani E. C., 2009). In order to overcome the difficulties of environmental quality assessment and management, it is crucial to handle operationally vast arrays of measurement data regarding the parameters characterizing its geological, chemical, and biological properties. By using a broad range of mathematical techniques and computers, this may be done at an appropriate level in line with today's standards (Kachiashvili K. J., 2009). Computer models that can forecast the movement of contaminants in natural water systems have become an urgent need in recent years because of the growing environmental concerns. In contrast to actual models, these digital representations have the advantage of being less expensive and more easily customized to meet the needs of the user. A new generation of tidal river pollution models can only be developed because of the broad use of mathematical modelling tools for studying hydrodynamics and pollutant transport (Nassehi V., 1993).

The Fickian technique is commonly employed in water quality research, particularly when it is necessary to get precise information on the distribution of pollutants in the water body (Ani E. C. C. V., 2012). Numerical simulation is used to create a one-dimensional water-quality model in a river. The boundary conditions of the governing equation are not uniform. Saulyev finite difference estimates pollutant concentrations (Samalerk P., 2018). For the temporal integration of parabolic equations, the cubic C^1 -spline collocation technique is an A-stable approach. In terms of both space and time, the suggested technique is accurate to the fourth order (Mohebbi A., 2010). Several different numerical approaches are going to be used in order to solve the one-dimensional advection–diffusion equation that has a constant coefficient. Approximations based on the finite difference at two levels are used in these various approaches (Dehghan M., 2004).

The proposed mathematical models and methods can be easily implemented using initial data, offering an efficient assessment of water quality in river systems. The model can be extended to other disciplines in science and engineering. It analyzes the distribution of polluting substances in rivers, considering multiple sources and types of pollutants. One-dimensional advection-diffusion models are created, hypothesizing the presence of pollution sources in the studied river section, including point or volume sources, underground sources, and other rivers flowing into the section.

The current investigation involves collecting water samples from the Tapi river in Surat, India. The purpose is to create a mathematical model that can forecast pollution levels at various distances and times. This work presents a mathematical model that describes the movement of pollutants in one dimension. The reduced differential transform method (RDTM) is used in order to solve the governing equation, which is a one-dimensional advection-diffusion equation. In addition to this, we have investigated the convergence of analytical solutions obtained by the RDTM method.

The mathematical expression for the problem is explicated in Section 2. A detailed summary of the foundational principles that form the basis of the RDTM is provided in Section 3. Section 4 contains the detailed process of converging the analytic series solution proposed by RDTM. As part of an evaluation of the method's efficacy, Section 5 examines the numerical results and convergence. A graphical representation of the obtained solutions is generated through the application of 3D plots. The conclusion is succinctly summarized in Section 6.

2 Mathematical Formulation of the problem

The Fickian diffusion equation can be used to represent the random distribution of particles in turbulent flow, according to Taylor's 1954 hypothesis. Diffusion in a channel flow is described by the one-dimensional differential equation (Kim S., 2006). In this section, a mathematical model of chemical transport, diffusion, and absorption is presented. After introducing the mathematical model, which is an advection-diffusion-reaction equation with appropriate initial and boundary conditions, different physical and geometrical parameters are then given (Kachiashvili K., 2007). Advection–diffusion equations must be solved in order to determine the pollutant concentration in the water sources. The following equation could be used to calculate concentration.

$$A\frac{\partial c}{\partial T}=\frac{\partial}{\partial X}\left(AD\frac{\partial c}{\partial X}\right)-Av^{*}\frac{\partial c}{\partial X}+Af \tag{1}$$

where $c\,(mg/L)$ is the BOD or COD concentration throughout its path through the medium along the flow field at any given time $T\,(hour)$ and distance $X\,(km)$. The Longitudinal Diffusion coefficient is expressed as $D\,(km^2/hour)$. The irregular uniform seepage flow velocity in the longitudinal direction is expressed as $v^{*}\,(km/hour)$. The flow area of a river's cross-section is $A\,(km^2)$. $f\,(mg/km^3hour)$ is a source term. X is the longitudinal distance from the study area's beginning. The dimension of time is T. Throughout the investigation, the cross-sectional area of the river is considered a constant. Equation (1) then transforms into the following form:

$$\frac{\partial c}{\partial T}=\frac{\partial}{\partial X}\left(D\frac{\partial c}{\partial X}\right)-v^{*}\frac{\partial c}{\partial X}+f \tag{2}$$

The description of the dimensionless variables is given as (Rubbab, 2016):

$$x=\frac{X}{L},\ t=\frac{DT}{L^2},\ v=\frac{v^{*}L}{D},\ S=\frac{L^2 f}{D} \tag{3}$$

L represents the length of the research section of the river. We obtain a dimensionless advection-diffusion equation in the given form.

$$\frac{\partial c}{\partial t}=\frac{\partial^2 c}{\partial x^2}-v\frac{\partial c}{\partial x}+S \tag{4}$$

To analyze this problem, we have used the concentration of pollutant substances NO_3 and PO_4 of river Tapi. In this paper, we have taken 5 km Research area of Tapi river from ONGC bridge to Pal Umra bridge. So here, river's length(L) is 5 km. Using collected data of pollutants NO_3 and PO_4 concentration, at time $t=0$ (any fixed time), the concentration is assumed as a polynomial form with good statistical indices.

In this article, the concentration level of contamination is discussed for both without and with various kind of input sources. Various cases have been examined to determine the concentration function's value and visual depiction. We're looking at two distinct sorts of known sources to see the pollution level in river. Here, S denotes the rate of the pollutant that enters the beginning border of the portion of the river that is being examined per unit of time and per unit of volume. According to the value of $S(x,t)$, these two types of input sources include exponential and sinusoidal source.

Initial function (5) for NO_3 pollutant concentration function (Tandel, 2024):

$$c(x,0) = -\frac{93x^5}{6250000} + \frac{1277x^4}{1562500} - \frac{17413x^3}{1250000} + \frac{3641x^2}{50000} + \frac{227x}{50000} + \frac{307}{500} \quad (5)$$

Initial function (6) for PO_4 pollutant concentration function (Tandel, 2024):

$$c(x,0) = -\frac{617x^5}{15625000} + \frac{3517x^4}{1562500} - \frac{53511x^3}{1250000} + \frac{1879x^2}{6250} - \frac{3157x}{6250} + \frac{2791}{1000} \quad (6)$$

3 Reduced Differential Transform Method

Let $b(\xi,\tau)$ be a two-variable function. Assume that $b(\xi,\tau)$ can be described as the combination of two single variable functions. i.e., $b(\xi,\tau) = k(\xi)l(\tau)$

By properties of differential transform, $b(\xi,\tau)$ can be represented as

$$b(\xi,\tau) = \sum_{i=0}^{\infty} M_{(i)}\xi^i \sum_{j=0}^{\infty} L_{(j)}\tau^j = \sum_{k=0}^{\infty} B_k(\xi)\tau^k$$

Where $B_k(\tau)$ is called t-dimensional spectrum function of $b(\xi,\tau)$ (Al-Amr, 2014).

$$B_k(\xi) = \frac{1}{k!}\left[\frac{\partial^k}{\partial\tau^k} b(\xi,\tau)\right]_{\tau=0} \quad (7)$$

In this paper, Uppercase letters $[B(\xi,\tau)]$ denote changed functions, whereas lowercase letters $[b(\xi,\tau)]$ denote original functions. The differential inverse transform of $B_k(\xi)$ is defined by

$$b(\xi,\tau) = \sum_{k=0}^{\infty} B_k(\xi)\tau^k \quad (8)$$

From Equations (7) and (8) We get

$$b(\xi,\tau) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial\tau^k} b(\xi,\tau)\right]_{\tau=0} \tau^k \quad (9)$$

Basic concept of RDTM, Consider the nonlinear differential partial differential equation:

$$Lb(\xi,\tau) + Rb(\xi,\tau) + Nb(\xi,\tau) = \vartheta(\xi,\tau) \quad (10)$$

with the initial condition $b(\xi,0) = y(x)$,

where $L = \frac{\partial}{\partial t}$, $Rb(\xi,\tau)$ is a linear operator which has partial derivatives, $Nb(\xi,\tau)$ is a nonlinear term and $\vartheta(\xi,\tau)$ is an inhomogeneous term.

By applying the transform on equation (10), we get

$$(k+1)B_{k+1}(\xi) = \psi_k(\xi) - RB_k(\xi) - NB_k(\xi) \quad (11)$$

where $B_k(\xi), \psi_k(\xi), RB_k(\xi)$ and $NB_k(\xi)$ are transform of $b(\xi,\tau), \vartheta(\xi,\tau), Rb(\xi,\tau)$ and $Nb(\xi,\tau)$ respectively. from initial condition, we can write

$$B_0(x) = y(x) \quad (12)$$

By solving equations (11) and (12), we get the values of $B_k(\xi)$. Subsequently, an estimated solution is generated by performing an inverse transformation on the collection of values $\{B_k(\xi)\}_{k=0}^{n}$. The result of this transformation is an approximate answer.

$$\tilde{b}_n(\xi,\tau) = \sum_{k=0}^{n} B_k(\xi)\tau^k \tag{13}$$

where n is an approximation order solution. Hence, the exact solution to the problem is stated by

$$b(\xi,\tau) = \lim_{n\to\infty} \tilde{b}_n(\xi,\tau). \tag{14}$$

Table 1 about here

4 Convergence Analysis

Theorem 4.1. If $\sum_{m=0}^{\infty} P_m t^m$ is given series (Maisuria M. A., 2023),

[A] $\exists\, 0<\delta<1 \ni \dfrac{\|P_{m+1}\|}{\|P_m\|} \le \delta \Rightarrow$ given series is convergent.

[B] $\exists\, \delta>1 \ni \dfrac{\|P_{m+1}\|}{\|P_m\|} \ge \delta \Rightarrow$ given series is divergent.

5 Results and discussion

5.1 NO_3 pollutant concentration

(a) Without pollutant source

In this case, $S(x,t)=0$ in equation (4). Using RDTM to solve equation (4) with initial condition (5). We get

$c(x,t) = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + \ldots$

where,

$$M_0 = -\frac{93x^5}{6250000} + \frac{1277x^4}{1562500} - \frac{17413x^3}{1250000} + \frac{3641x^2}{50000} + \frac{227x}{50000} + \frac{307}{500}$$

$$M_1 = \frac{93x^4}{2500000} - \frac{3019x^3}{1562500} + \frac{383787x^2}{12500000} - \frac{195503x}{1250000} + \frac{14337}{100000}$$

$$M_2 = -\frac{93x^3}{2500000} + \frac{2613x^2}{1562500} - \frac{528699x}{25000000} + \frac{1745089}{25000000}$$

$$M_3 = \frac{93x^2}{5000000} - \frac{3949x}{6250000} + \frac{231977}{50000000}$$

Here,

$$\frac{\|M_1\|}{\|M_0\|} = 0.2335 < 1, \frac{\|M_2\|}{\|M_1\|} = 0.4869 < 1, \frac{\|M_3\|}{\|M_2\|} = 0.0665 < 1$$

Given series solution of nitrate concentration function is converge to exact solution.

Table 2 about here

Table 2 give the numerical values of nitrate pollutant concentration without input source at different level of distance and time.

(b) With exponential input source

In this case, $S(x,t)=\dfrac{2e^{-t}}{(x+L)^3}$ in equation (4). Using RDTM to solve equation (4) with initial condition (5). We get

$c(x,t)=H_0+H_1t+H_2t^2+H_3t^3+\ldots$

where,

$$H_0=-\frac{93x^5}{6250000}+\frac{1277x^4}{1562500}-\frac{17413x^3}{1250000}+\frac{3641x^2}{50000}+\frac{227x}{50000}+\frac{307}{500}$$

$$H_1=\frac{2}{(x+5)^3}-\frac{195503x}{1250000}+\frac{383787x^2}{12500000}-\frac{3019x^3}{1562500}+\frac{93x^4}{2500000}+\frac{14337}{100000}$$

$$H_2=\frac{2613x^2}{1562500}-\frac{2x^2+17x+11}{2(x+5)^5}-\frac{528699x}{25000000}-\frac{93x^3}{2500000}+\frac{1745089}{25000000}$$

$$H_3=\frac{2x^4+37x^3+237x^2+715x+1745}{6(x+5)^7}-\frac{3949x}{6250000}+\frac{93x^2}{5000000}+\frac{231977}{50000000}$$

Here,

$$\frac{\|H_1\|}{\|H_0\|}=0.2596<1,\ \frac{\|H_2\|}{\|H_1\|}=0.4270<1,\ \frac{\|H_3\|}{\|H_2\|}=0.1229<1$$

Therefore, given series solution is converge to exact solution.

Table 3 about here

The values of nitrate pollutant concentration at various values of x and t with exponential input pollutant source are presented in table 3.

(c) With sinusoidal input source

In this case, $S(x,t)=\dfrac{2\sin t}{(x+L)^3}$ in equation (4). Using RDTM to solve equation (4) with initial condition (5). We get

$c(x,t)=D_0+D_1t+D_2t^2+D_3t^3+\ldots$

where,

$$D_0=-\frac{93x^5}{6250000}+\frac{1277x^4}{1562500}-\frac{17413x^3}{1250000}+\frac{3641x^2}{50000}+\frac{227x}{50000}+\frac{307}{500}$$

$$D_1=\frac{93x^4}{2500000}-\frac{3019x^3}{1562500}+\frac{383787x^2}{12500000}-\frac{195503x}{1250000}+\frac{14337}{100000}$$

$$D_2 = \frac{1}{(x+5)^3} - \frac{528699x}{25000000} + \frac{2613x^2}{1562500} - \frac{93x^3}{2500000} + \frac{1745089}{25000000}$$

$$D_3 = \frac{x+13}{2(x+5)^5} - \frac{3949x}{6250000} + \frac{93x^2}{5000000} + \frac{231977}{50000000}$$

Here,

$$\frac{\|D_1\|}{\|D_0\|} = 0.2335 < 1, \frac{\|D_2\|}{\|D_1\|} = 0.5427 < 1, \frac{\|D_3\|}{\|D_2\|} = 0.0864 < 1$$

Therefore, given series solution is converge to exact solution.

Table 4 about here

The values of nitrate pollutant concentration at various values of x and t with sinusoidal input pollutant source are presented in table 4.

Figure 1 about here

Figure 1 give the 3D graphical comparison of solution function obtained for all cases.

5.2 Phosphate concentration

(a) Without pollutant source

In this case, $S(x,t) = 0$ in equation (4). Using RDTM to solve equation (4) with initial condition (6). We get

$c(x,t) = A_0 + A_1 t + A_2 t^2 + A_3 t^3 + ...$

where,

$$A_0 = -\frac{617x^5}{15625000} + \frac{3517x^4}{1562500} - \frac{53511x^3}{1250000} + \frac{1879x^2}{6250} - \frac{3157x}{6250} + \frac{2791}{1000}$$

$$A_1 = \frac{617x^4}{6250000} - \frac{2067x^3}{390625} + \frac{1140297x^2}{12500000} - \frac{348433x}{625000} + \frac{10673}{12500}$$

$$A_2 = -\frac{617x^3}{6250000} + \frac{14253x^2}{3125000} - \frac{1537161x}{25000000} + \frac{1441231}{6250000}$$

$$A_3 = \frac{617x^2}{12500000} - \frac{671x}{390625} + \frac{664419}{50000000}$$

Here,

$$\frac{\|A_1\|}{\|A_0\|} = 0.3059 < 1, \frac{\|A_2\|}{\|A_1\|} = 0.2701 < 1, \frac{\|A_3\|}{\|A_2\|} = 0.0576 < 1$$

Given series solution of NO3 concentration function is converge to exact solution.

Table 5 about here

Table 2 give the numerical values of nitrate pollutant concentration without input source at different level of distance and time.

(b) With exponential input source

In this case, $S(x,t)=\dfrac{2e^{-t}}{(x+L)^3}$ in equation (4). Using RDTM to solve equation (4) with initial condition (6). We get

$c(x,t)=B_0+B_1t+B_2t^2+B_3t^3+...$

where,

$$B_0=-\frac{617x^5}{15625000}+\frac{3517x^4}{1562500}-\frac{53511x^3}{1250000}+\frac{1879x^2}{6250}-\frac{3157x}{6250}+\frac{2791}{1000}$$

$$B_1=\frac{2}{(x+5)^3}-\frac{348433x}{625000}+\frac{1140297x^2}{12500000}-\frac{2067x^3}{390625}+\frac{617x^4}{6250000}+\frac{10673}{12500}$$

$$B_2=\frac{14253x^2}{3125000}-\frac{2x^2+17x+11}{2(x+5)^5}-\frac{1537161x}{25000000}-\frac{617x^3}{6250000}+\frac{1441231}{6250000}$$

$$B_3=\frac{2x^4+37x^3+237x^2+715x+1745}{(x+5)^7}-\frac{671x}{390625}+\frac{617x^2}{12500000}+\frac{664419}{50000000}$$

Here,

$$\frac{\|B_1\|}{\|B_0\|}=0.3117<1,\ \frac{\|B_2\|}{\|B_1\|}=0.2631<1,\ \frac{\|B_3\|}{\|B_2\|}=0.0743<1$$

Therefore, given series solution is converge to exact solution.

Table 6 about here

The values of nitrate pollutant concentration at various values of x and t with exponential input pollutant source are presented in table 3.

(c) With sinusoidal input source

In this case, $S(x,t)=\dfrac{2\sin t}{(x+L)^3}$ in equation (4). Using RDTM to solve equation (4) with initial condition (6). We get

$c(x,t)=C_0+C_1t+C_2t^2+C_3t^3+...$

where,

$$C_0=-\frac{617x^5}{15625000}+\frac{3517x^4}{1562500}-\frac{53511x^3}{1250000}+\frac{1879x^2}{6250}-\frac{3157x}{6250}+\frac{2791}{1000}$$

$$C_1=\frac{617x^4}{6250000}-\frac{2067x^3}{390625}+\frac{1140297x^2}{12500000}-\frac{348433x}{625000}+\frac{10673}{12500}$$

$$C_2 = \frac{1}{(x+5)^3} - \frac{1537161x}{25000000} + \frac{14253x^2}{3125000} - \frac{617x^3}{6250000} + \frac{1441231}{6250000}$$

$$C_3 = \frac{x+13}{2(x+5)^5} - \frac{671x}{390625} + \frac{617x^2}{12500000} + \frac{664419}{50000000}$$

Here,

$$\frac{\|C_1\|}{\|C_0\|} = 0.3059 < 1, \frac{\|C_2\|}{\|C_1\|} = 0.2794 < 1, \frac{\|C_3\|}{\|C_2\|} = 0.0644 < 1$$

Therefore, given series solution is converge to exact solution.

Table 7 about here

The values of nitrate pollutant concentration at various values of x and t with sinusoidal input pollutant source are presented in table 4.

Figure 2 about here

Figure 2 give the 3D graphical comparison of solution function obtained for all cases.

6 Conclusion

The current problem was studied under two scenarios: no input source and input source. This work aims to detect and reduce exponential and sinusoidal input pollution. To increase input data quality and dependability, offer appropriate solutions for tackling various forms of contamination. This study examines exponential and sinusoidal input pollution sources to propose solutions to reduce their negative impacts. We found that exponential input sources had higher concentrations than sinusoidal input sources at different distances and periods. In particular, when the input source is sinusoidal, concentration is greater than when it is absent over distances and time intervals. Our study shows that input sources increase pollution levels. This work solves a one-dimensional advection-diffusion equation with a source term using the Reduced Differential Transform Method (RDTM). Data is fitted to a polynomial function to establish the starting condition. This method may be extended to two- and three-dimensional advection-diffusion equations.

Acknowledgements

Priti V. Tandel is thankful to Veer Narmad South Gujarat University, Surat for providing the opportunity to undertake the minor project. This research is conducted as part of a minor project at Veer Narmad South Gujarat University, Surat and the experience gained has been invaluable to academic and professional development.

References

Al-Amr, M. O. (2014). New applications of reduced differential transform method. *Alexandria Engineering Journal*, 243-247.

Ani E. C., C. V. (2012). Mathematical models to support pollution counteraction in case of accidents. *Environmental Engineering & Management Journal (EEMJ), 11*(1). Retrieved from http://eemj.eu/index.php/EEMJ/article/view/981

Ani E. C., W. S. (2009). Development, calibration and evaluation of two mathematical models for pollutant transport in a small river. *Environmental Modelling & Software, 24*(10), 1139-1152. doi:https://doi.org/10.1016/j.envsoft.2009.03.008

Kachiashvili K. J., a. M. (2009). Software realization problems of mathematical models of pollutants transport in rivers. *Advances in Engineering Software, 40*(10), 1063-1073. doi:https://doi.org/10.1016/j.advengsoft.2009.03.016

Kachiashvili K., G. D. (2007). Modeling and simulation of pollutants transport in rivers. *Applied mathematical modelling, 31*(7), 1371-1396. doi:https://doi.org/10.1016/j.apm.2006.02.015

Keskin, Y. &. (2010). Reduced differential transform method for generalized KdV equations. *Mathematical and Computational applications*, 382-393.

Kim S., a. K. (2006). Generalized Fick's law and fractional ADE for pollution transport in a river: detailed derivation. *Journal of Hydrologic Engineering, 11*(1), 80-83. doi:https://doi.org/10.1061/(ASCE)1084-0699(2006)11:1(80)

M., D. (2004). Weighted finite difference techniques for the one-dimensional advection–diffusion equation. *Applied Mathematics and Computation, 147*(2), 307-319. doi:https://doi.org/10.1016/S0096-3003(02)00667-7

Maisuria, M. A. (2023). Mathematical modelling and application of reduced differential transform method for river pollution. *Ratio Mathematica*.

Maisuria, M. A. (2023). Solution of Two-Dimensional Solute Transport Model for Heterogeneous Porous Medium Using Fractional Reduced Differential Transform Method. *Axioms, 12*(11), 1039.

Mohebbi A., a. D. (2010). High-order compact solution of the one-dimensional heat and advection–diffusion equations. *Applied mathematical modelling, 34*(10), 3071-3084. doi:https://doi.org/10.1016/j.apm.2010.01.013

Nassehi V., a. B. (1993). A mathematical model for the hydrodynamics and pollutants transport in long and narrow tidal rivers. *Applied mathematical modelling, 17*(8), 415-422. doi:https://doi.org/10.1016/0307-904X(93)90116-X

Rubbab, Q. M. (2016). Analytical solutions to the fractional advection-diffusion equation with time-dependent pulses on the boundary. *AIP Advances, 6*, 075318(1-11).

Samalerk P., a. P. (2018). Numerical simulation of a one-dimensional water-quality model in a stream using a Saulyev technique with quadratic interpolated initial-boundary conditions. *In Abstract and applied analysis, 2018*, 1926519. doi:https://doi.org/10.1155/2018/1926519

Srivastava, V. K. (2014). Analytical approximations of two and three dimensional time-fractional telegraphic equation by reduced differential transform method. . *Egyptian Journal of Basic and Applied Sciences*, 60-66.

Tandel, P. V. (2024). Numerical solution of pollutant transport model with source term by RDTM. *Ratio Mathematica, 51*.

Table 1: Reduced differential transformation (Keskin, 2010; Maisuria M. A., 2023; Srivastava, 2014)

Original form	Transformed form
$b(\xi,\tau)$	$B_k(\xi)=\frac{1}{k!}\left[\frac{\partial^k}{\partial\tau^k}b(\xi,\tau)\right]_{\tau=0}$
$\alpha p(\xi,\tau)\pm\beta q(\xi,\tau)$	$\alpha P_k(\xi)\pm\beta Q_k(\xi)$
$\xi^m\tau^n$	$\xi^m\delta(k-n)$
$\xi^m\tau^n b(\xi,\tau)$	$\xi^m B_{k-n}(\xi)$
$z(\xi,\tau)=p(\xi,\tau)q(\xi,\tau)$	$Z_k(\xi)=\sum_{r=0}^{k}P_r(\xi)Q_{k-r}(\xi)$
$\frac{\partial^r}{\partial\tau^r}b(\xi,\tau)$	$\frac{(k+r)!}{k!}B_{k+r}(\xi)$
$\frac{\partial}{\partial\xi}b(\xi,\tau)$	$\frac{\partial}{\partial\xi}B_k(\xi)$

Table 2: Numerical value of nitrate at different x and t

$x\backslash t$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.15	0.629003	0.643095	0.658574	0.675468	0.693805	0.713612	0.734915	0.757743	0.782123	0.808081
0.25	0.630738	0.643324	0.657256	0.672561	0.689266	0.707398	0.726984	0.748051	0.770625	0.794733
0.35	0.633786	0.644926	0.657371	0.671147	0.686282	0.702802	0.720733	0.740101	0.760934	0.783258
0.45	0.638067	0.647819	0.658836	0.671144	0.684769	0.699737	0.716075	0.733808	0.752963	0.773566
0.55	0.643504	0.651926	0.661573	0.672471	0.684646	0.698123	0.712928	0.729086	0.746625	0.765569
0.65	0.650023	0.657171	0.665505	0.67505	0.685832	0.697876	0.711208	0.725853	0.741836	0.759183
0.75	0.657551	0.66348	0.670556	0.678805	0.688251	0.69892	0.710837	0.724026	0.738513	0.754323
0.85	0.666015	0.670779	0.676652	0.683659	0.691826	0.701176	0.711735	0.723527	0.736577	0.750909
0.95	0.675347	0.678998	0.683722	0.689542	0.696484	0.704571	0.713828	0.724279	0.735948	0.748861

Table 3: Numerical value of nitrate at different x and t

$x\backslash t$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.15	0.630452	0.645974	0.662885	0.681231	0.70106	0.722418	0.745351	0.769906	0.79613	0.824069
0.25	0.632104	0.646035	0.661308	0.67797	0.696063	0.715632	0.736723	0.759379	0.783646	0.809567
0.35	0.635075	0.647481	0.661185	0.67623	0.692659	0.710513	0.729837	0.750673	0.773062	0.797049
0.45	0.639285	0.65023	0.662431	0.675927	0.69076	0.706971	0.724601	0.74369	0.76428	0.786411
0.55	0.644657	0.654205	0.664966	0.676978	0.690282	0.704918	0.720923	0.738338	0.757203	0.777556
0.65	0.651114	0.659326	0.66871	0.679302	0.691143	0.704268	0.718718	0.734529	0.751739	0.770388
0.75	0.658585	0.66552	0.673586	0.682821	0.69326	0.704941	0.7179	0.732174	0.7478	0.764814

0.85	0.666996	0.672713	0.679521	0.687457	0.696557	0.706855	0.718388	0.73119	0.745298	0.760747
0.95	0.676278	0.680833	0.686441	0.693137	0.700956	0.709933	0.720101	0.731496	0.744151	0.758101

Table 4: Numerical value of nitrate at different x and t

$x \backslash t$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.15	0.629079	0.643402	0.659282	0.676756	0.695862	0.716639	0.739125	0.763358	0.789376	0.817217
0.25	0.630809	0.643614	0.657923	0.673773	0.691202	0.710245	0.73094	0.753324	0.777433	0.803305
0.35	0.633852	0.645199	0.657999	0.67229	0.688105	0.705482	0.724455	0.745061	0.767334	0.791311
0.45	0.63813	0.648077	0.65943	0.672222	0.686488	0.702263	0.719582	0.738478	0.758987	0.781142
0.55	0.643564	0.652171	0.662135	0.67349	0.686269	0.700506	0.716235	0.733489	0.752301	0.772705
0.65	0.65008	0.657403	0.666036	0.676013	0.687367	0.700128	0.714331	0.730008	0.747191	0.765913
0.75	0.657604	0.663699	0.671059	0.679716	0.689703	0.70105	0.713789	0.727953	0.743571	0.760677
0.85	0.666066	0.670987	0.677129	0.684523	0.693201	0.703193	0.714529	0.727241	0.74136	0.756915
0.95	0.675395	0.679195	0.684174	0.690361	0.697787	0.706482	0.716475	0.727796	0.740476	0.754544

Table 5: Numerical value of phosphate at different x and t

$x \backslash t$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.15	2.801306	2.885267	2.973813	3.067024	3.164977	3.26775	3.375422	3.488071	3.605775	3.728612
0.25	2.757026	2.83559	2.918619	3.006189	3.098378	3.195263	3.296921	3.40343	3.514866	3.631307
0.35	2.71831	2.791655	2.869344	2.951455	3.038063	3.129244	3.225075	3.325631	3.430989	3.541226
0.45	2.684913	2.753213	2.825739	2.902568	2.983774	3.069433	3.159619	3.254409	3.353876	3.458097
0.55	2.656597	2.720022	2.787559	2.85928	2.935261	3.015576	3.100298	3.189502	3.283263	3.381653
0.65	2.633128	2.691847	2.754563	2.821349	2.892278	2.967424	3.046859	3.130656	3.21889	3.311633
0.75	2.614278	2.668456	2.726518	2.788537	2.854584	2.924733	2.999054	3.077621	3.160505	3.247779
0.85	2.599824	2.649622	2.703194	2.760611	2.821944	2.887264	2.956643	3.030152	3.107861	3.189842
0.95	2.589547	2.635124	2.684367	2.737345	2.794127	2.854785	2.919389	2.988009	3.060714	3.137576

Table 6: Numerical value of phosphate at different x and t

$x \backslash t$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.15	2.802755	2.888146	2.978124	3.072787	3.172231	3.276556	3.385857	3.500234	3.619782	3.7446
0.25	2.758392	2.838301	2.922671	3.011597	3.105174	3.203497	3.30666	3.414758	3.527887	3.646141
0.35	2.719599	2.79421	2.873159	2.956537	3.044439	3.136955	3.234179	3.336202	3.443117	3.555017
0.45	2.686132	2.755624	2.829334	2.907351	2.989765	3.076666	3.168145	3.26429	3.365193	3.470943
0.55	2.65775	2.722301	2.790951	2.863787	2.940898	3.022371	3.108294	3.198754	3.29384	3.39364
0.65	2.634219	2.694002	2.757768	2.825601	2.897588	2.973816	3.054368	3.139332	3.228793	3.322837
0.75	2.615312	2.670496	2.729549	2.792553	2.859593	2.930754	3.006117	3.085769	3.169792	3.25827
0.85	2.600805	2.651556	2.706063	2.764409	2.826675	2.892943	2.963296	3.037815	3.116582	3.19968
0.95	2.590478	2.636959	2.687086	2.74094	2.7986	2.860148	2.925663	2.995225	3.068916	3.146815

Table 7: Numerical value of phosphate at different x and t

x\t	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.15	2.801381	2.885574	2.974521	3.068311	3.167034	3.270778	3.379632	3.493686	3.613028	3.737748
0.25	2.757097	2.83588	2.919286	3.007401	3.100313	3.198109	3.300877	3.408703	3.521674	3.639879
0.35	2.718377	2.791928	2.869973	2.952597	3.039885	3.131924	3.228797	3.33059	3.437389	3.549279
0.45	2.684977	2.753471	2.826333	2.903646	2.985493	3.071959	3.163126	3.259079	3.3599	3.465673
0.55	2.656657	2.720267	2.78812	2.860298	2.936884	3.017959	3.103606	3.193905	3.288939	3.388789
0.65	2.633185	2.692079	2.755094	2.822312	2.893812	2.969676	3.049982	3.134812	3.224245	3.318363
0.75	2.614332	2.668675	2.727021	2.789448	2.856036	2.926863	3.002007	3.081548	3.165563	3.254133
0.85	2.599875	2.64983	2.703671	2.761475	2.823319	2.889281	2.959437	3.033866	3.112644	3.195848

0.95	2.589595	2.635322	2.68482	2.738164	2.795431	2.856697	2.922036	2.991526	3.065241	3.143258

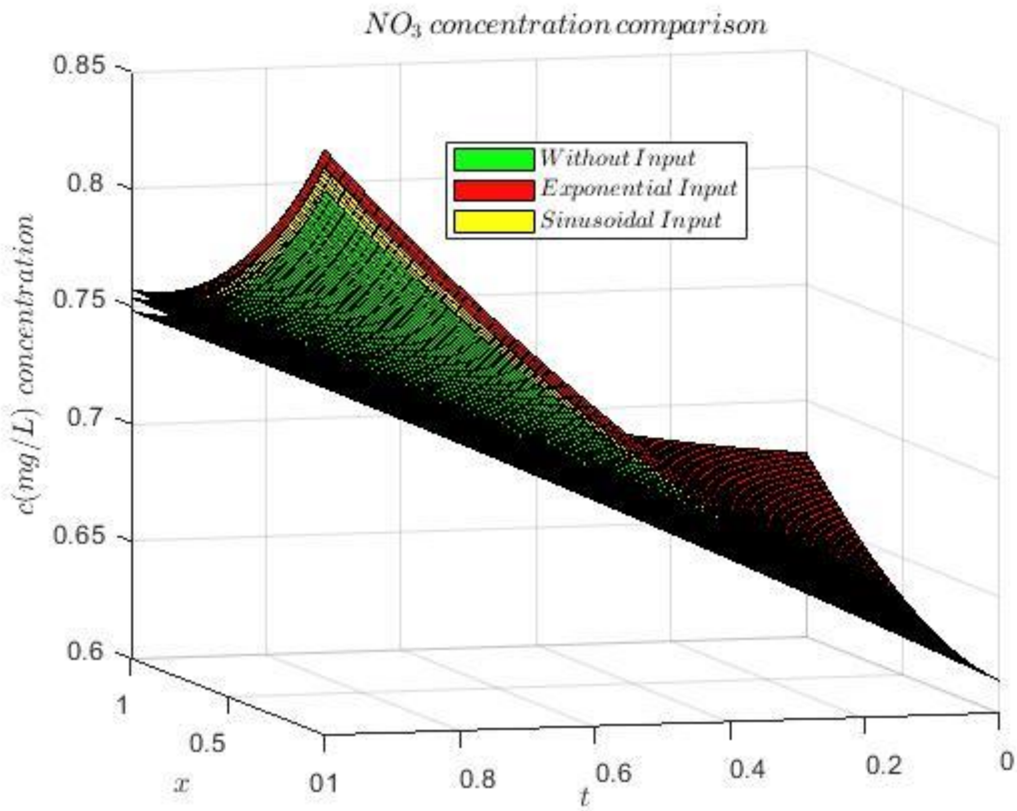

Figure 1: 3D graph comparison of nitrate concentration function

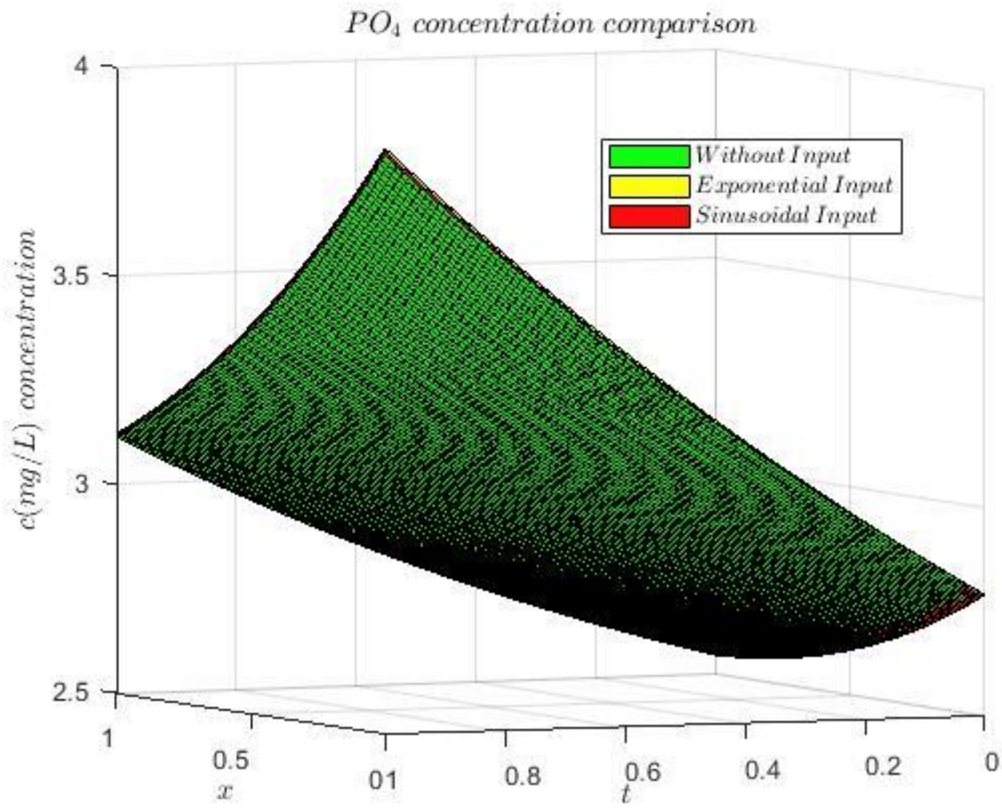

Figure 2: 3D graph comparison of phosphate concentration function